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Apex Base Least Cost Method For Fuzzy Transportation Problem

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Abstract– A new approach to solve the fuzzy transportation problem using generalized trapezoidal fuzzy numbers is proposed. A new ranking method for generalized trapezoidal fuzzy number is introduced and constructed. A numerical example is solved using the proposed algorithm.

Keywords– Generalized trapezoidal fuzzy numbers, Apex and base angles, Ranking function, Least adjacent cost method.

1 INTRODUCTION

Transportation problem was originally developed by Tolstoi, it can be formulated as a linear programming problem and its optimal solution can be obtained using simplex method. Charnes and Cooper [4] developed the stepping stone method which provides an alternative way of determining the optimal solution. After the entry of the new notion "Fuzzy Sets" introduced by L. A. Zadeh [8] in 1965, the decision making problems were all fuzzified, because in real life situations all are uncertain or fuzzy. So came the existence of fuzzy transportation problem (FTP) which is a transportation problem (TP) in which the supply and demand are fuzzy quantities. Most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. Shiang-Tai Liu and Chiang Kao [7], Chanas et al. [3], Chanas and Kuchta [2], proposed a method for solving fuzzy transportation problem. Nagoor Gani and Abdul Rezak [5] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Pandian et al. [6], proposed a method namely, zero point method, for finding a fuzzy optimal solution for a fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. Amarpreet kaur[1] proposed a new method for solving fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of the transportation cost, availability and demand of the product. In many fuzzy decision problems, the data are represented in terms of fuzzy numbers.

In this paper, we propose a new method named as "Apex Base Least Cost method"(ABLC Method) to solve a fuzzy transportation problem with generalized trapezoidal fuzzy numbers... Here we have proposed a new method for ranking the generalized trapezoidal fuzzy numbers based on apex and base angles. This method provides the correct ordering of generalized trapezoidal fuzzy numbers, and this proposed method is illustrated with the help of numerical example.

2 PRELIMINARIES

Zadeh [2] in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

2.1 Definition

For the universal set A, a function $\mu_A(x)$ from A to [0,1] is called the membership function. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined as $A^* = \{(x, \mu_A(x): x \in A \ and\ \mu_A(x) \in \epsilon[0,1])\}$.

2.2 Definition

A fuzzy set A^*, defined on the universal set of real numbers R is said to be a generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{A^*}: R \rightarrow [0, \omega]$ is continuous.
2. $\mu_{A^*}(x) = 0$ For all $x\, \epsilon(-\infty, a]\ \mathrm{U}\ [d, \infty)$.
3. $\mu_{A^*}(x)$ is strictly increasing on $[a, b]$and strictly decreasing on [c, d].
4. $\mu_{A^*}(x) = \omega$, for all $x \epsilon [b, c]$, where $0 < \omega \leq 1$.

2.3 Definition

A generalized fuzzy number $A^* = (a, b, c, d; \omega)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{A^*}(x) = \begin{cases} \omega\left(\dfrac{x-a}{b-a}\right), & a \leq x \leq b \\ \omega, & b \leq x \leq c \\ \omega\left(\dfrac{d-x}{d-c}\right), & c \leq x \leq d \\ 0, & otherwise \end{cases}$$

3 FUZZY AGGREGATION

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number.

Geometric Mean

The geometric mean aggregation operator defined on n trapezoidal fuzzy numbers

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$(a_1,b_1,c_1,d_1),(a_2,b_2,c_2,d_2),\ldots\ldots\ldots,(a_n,b_n,c_n,d_n)$ is again a trapezoidal fuzzy number (a,b,c,d) where

$$a=\left(\prod_1^n a_i\right)^{1/n},\qquad b=\left(\prod_1^n b_i\right)^{1/n},$$
$$c=\left(\prod_1^n c_i\right)^{1/n}\ and\ \ d=\left(\prod_1^n d_i\right)^{1/n}.$$

Consider the trapezoidal fuzzy number shown in Figure 1.If the value of this trapezoidal fuzzy number is $v\in[b,c]$with the corresponding possibility $\mu=1$. The left side apex angle and base angle of this trapezoidal fuzzy number is $\angle apb$ and $\angle pab$ respectively. The right side apex angle and base angle of this trapezoidal fuzzy number is $\angle drc$ and $\angle rdc$ respectively. The left and right side apex angles are subtended to the left and right of the interval $[b,c]$ respectively.

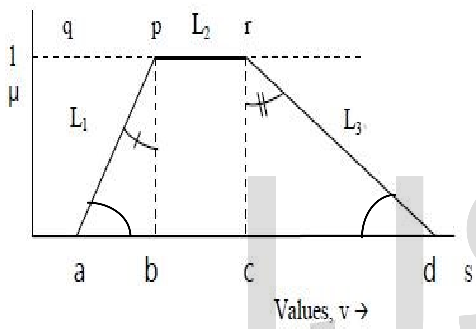

Fig. 1 Trapezoidal Fuzzy Numbers

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But, $\angle apb=\frac{\pi}{2}-\angle bap$ (Left apex)

$\angle pab=\frac{\pi}{2}-\angle apb$ (Left base)

Considering the left side and geometric mean over n trapezoidal fuzzy numbers we have,

$$\left(\prod_1^n \angle apb_i\right)^{1/n}=\left(\prod_1^n \angle\left(\frac{\pi}{2}-bap\right)_i\right)^{1/n}$$
$$=\frac{\pi}{2}-\left(\prod_1^n \angle bap_i\right)^{1/n}$$

The left side of the above equation represents the contributions of the left lines (aggregate apex angle). It can be seen that

$$tan\left[\prod_1^n \angle apb_i\right]^{1/n}=tan\left[\frac{\pi}{2}-\prod_1^n \angle\, bap_i\right]^{1/n}$$
$$=cot\left(\prod_1^n \angle\, bap_i\right)^{1/n}$$
$$=\frac{1}{tan\left[\prod_1^n \angle\, bap_i\right]^{1/n}}$$

Also it can be shown that

$$tan\left[\prod_1^n \angle apb_i\right]^{1/n}=tan\left[\prod_1^n \tan^{-1} tan\angle\, apb_i\right]^{1/n}$$
$$=tan\left[\prod_1^n \tan^{-1}(b_i-a_i)\right]^{1/n}\qquad(1)$$

and

$$tan\left[\prod_1^n \angle apb_i\right]^{1/n}\ =\ b-a\qquad(2)$$

from the equation (1) and (2) we have,

$$b-a=\ tan\left[\prod_1^n \tan^{-1}(b_i-a_i)\right]^{1/n}$$

$$a=b-\ tan\left[\prod_1^n \tan^{-1}(b_i-a_i)\right]^{1/n}$$

Under identical treatment, it can be shown that

$$b=\left(\prod_1^n b_i\right)^{1/n}$$

Subsequently it is possible to show that

$$a=\left(\prod_1^n b_i\right)^{1/n}-\ tan\left[\prod_1^n \tan^{-1}(b_i-a_i)\right]^{1/n}$$
$$b=\left(\prod_1^n a_i\right)^{1/n}+\ tan\left[\prod_1^n \tan^{-1}(b_i-a_i)\right]^{1/n}$$
$$c=\left(\prod_1^n d_i\right)^{1/n}-\ tan\left[\prod_1^n \tan^{-1}(d_i-c_i)\right]^{1/n} and$$
$$d=\left(\prod_1^n c_i\right)^{1/n}+\ tan\left[\prod_1^n \tan^{-1}(d_i-c_i)\right]^{1/n}$$

4 RANKING METHOD

In this section, a new approach is proposed for the ranking of generalized trapezoidal fuzzy numbers and using the proposed approach, in the theorem.

Definition 4.1

Let $\mathrm{A}=(\mathrm{a}_1,\mathrm{b}_1,\mathrm{c}_1,\mathrm{d}_1;\omega_1)$ and $\mathrm{B}=(\mathrm{a}_2,\mathrm{b}_2,\mathrm{c}_2,\mathrm{d}_2;\omega_2)$ be two generalized trapezoidal fuzzy numbers then the ranking function R satisfies the following properties

$\mathrm{A}>\mathrm{B}$ if $\mathrm{R(A)}>\mathrm{R(B)}$

$\mathrm{A}<\mathrm{B}$ if $\mathrm{R(A)}<\mathrm{R(B)}$

$\mathrm{A}\approx\mathrm{B}$ if $\mathrm{R(A)}\approx\mathrm{R(B)}$

4.2 Method to find the values of R (A) and R (B)

Let $\mathrm{A}=(\mathrm{a}_1,\mathrm{b}_1,\mathrm{c}_1,\mathrm{d}_1;\omega_1)$ and $\mathrm{B}=(\mathrm{a}_2,\mathrm{b}_2,\mathrm{c}_2,\mathrm{d}_2;\omega_2)$ be two generalized trapezoidal fuzzy numbers, the following steps are used to find the values of R (A) and R (B).

Step: 1

Find $\mathrm{b}_1,\mathrm{b}_1$and $\mathrm{c}_1,\mathrm{c}_2$ where

$$b_1=\omega_1\left[\prod_1^n b_{1_i}\right]^{1/n},\qquad b_2=\omega_2\left[\prod_1^n b_{2_i}\right]^{1/n},$$
$$c_1\ =\omega_1\left[\prod_1^n c_{1_i}\right]^{1/n} and\ \ c_2=\omega_2\left[\prod_1^n c_{2_i}\right]^{1/n}$$

Step: 2

Find $a_1,a_1$and d_1,d_2 where

$$a_1\ =\omega_1\left[\prod_1^n a_{1_i}\right]^{1/n},a_2=\omega_2\left[\prod_1^n a_{2_i}\right]^{1/n}$$
$$d_1\ =\omega_1\left[\prod_1^n d_{1_i}\right]^{1/n} and\ d_2=\omega_2\left[\prod_1^n d_{2_i}\right]^{1/n}$$

Step: 3

Find $p(A),q(A),r(A)\ and\ s(A)$ where

$$p(A)=a_1=\omega_1\left[\prod_1^n b_{1_i}\right]^{1/n}-\tan\left[\omega_1\prod_1^n tan^{-1}\left(b_{1_i}-a_{1_i}\right)\right]^{1/n}$$
$$q(A)=b_1=\omega_1\left[\prod_1^n a_{1_i}\right]^{1/n}+\tan\left[\omega_1\prod_1^n tan^{-1}\left(b_{1_i}-a_{1_i}\right)\right]^{1/n}$$
$$r(A)=c_1\ =\omega_1\left[\prod_1^n d_{1_i}\right]^{1/n}-\tan\left[\omega_1\prod_1^n tan^{-1}\left(d_{1_i}-c_{1_i}\right)\right]^{1/n}$$
$$s(A)=d_1\ =\omega_1\left[\prod_1^n c_{1_i}\right]^{1/n}+\tan\left[\omega_1\prod_1^n tan^{-1}\left(d_{1_i}-c_{1_i}\right)\right]^{1/n}$$

Step 4:

Find $p(B),q(B),r(B)\ and\ s(B)$ where

$$p(B)=a_2=\omega_2\left[\prod_1^n b_{2_i}\right]^{1/n}-\tan\left[\omega_2\prod_1^n tan^{-1}\left(b_{2_i}-a_{2_i}\right)\right]^{1/n}$$
$$q(B)=b_2=\omega_2\left[\prod_1^n a_{2_i}\right]^{1/n}+\tan\left[\omega_2\prod_1^n tan^{-1}\left(b_{2_i}-a_{2_i}\right)\right]^{1/n}$$
$$r(B)=c_2\ =\omega_2\left[\prod_1^n d_{2_i}\right]^{1/n}-\tan\left[\omega_2\prod_1^n tan^{-1}\left(d_{2_i}-c_{2_i}\right)\right]^{1/n}$$

$s(B) = d_2 = \omega_2\left[\prod_1^n c_{2_i}\right]^{1/n} + \tan\left[\omega_2 \prod_1^n \tan^{-1}\left(d_{2_i} - c_{2_i}\right)\right]^{1/n}$

Step: 5

Find $R(A)$ and $R(B)$ defined by,

$R(A) = \sqrt{\frac{p(A)^2+q(A)^2+r(A)^2+s(A)^2}{4}}$ and

$R(B) = \sqrt{\frac{p(B)^2+q(B)^2+r(B)^2+s(B)^2}{4}}$

This is known as Root Average of Sum Square.

4.2 Example

Let A=(2,3,6,8;0.5) and B=(2,6,11,13;0.5) be two generalized trapezoidal fuzzy numbers then

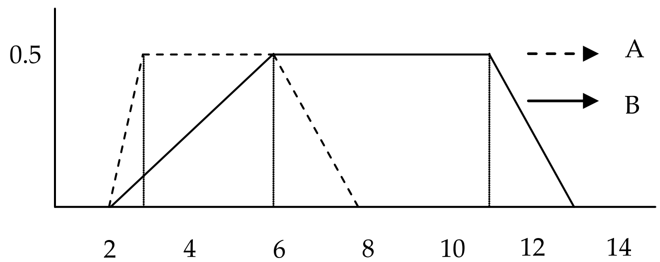

Fig. 2 Representation of GTFNs

$b_A = 1.5$	$b_B = 3$	$c_A = 3$	$c_B = 5.5$
$a_A = 1$	$a_B = 1$	$d_A = 4$	$d_B = 6.5$

and

$p(A) = 1.089$ $P(B) = 2.219$
$q(A) = 1.411$ $q(B) = 1.780$
$r(A) = 3.383$ $r(B) = 5.88$
$s(A) = 3.617$ $s(B) = 6.118$

and $R(A) = 2.63$ $R(B) = 4.4$

$i.e.,$ $R(A) < R(B)$

Hence $A < B$

5 Apex Base Least Cost Method (ABLC Method)

In this section, a new method (ABLC) is proposed to find a fuzzy initial basic feasible solution using ranking function, in which transportation costs are represented as generalized trapezoidal fuzzy numbers.

Algorithm

Step 1 Construct the fuzzy transportation table from the given fuzzy transportation problem.

Step 2 Convert the transportation table into a balanced one, if it is not.

Step 3 Select the row or column corresponding to the least demand or supply.

Step 4 Select a cell in a marked row/or column with minimum cost. If tie occurs, choose the cell with the least average value of its fuzzy supply and demand.

Step 5 Find the adjacent row cost and adjacent column cost for the selected cell in the selected row/or column and choose the adjacent cell with minimum cost. Allocate the corresponding minimum of its supply and demand.

Step 6 After performing Step 5 delete the row or column (where supply or demand becomes zero) for further calculation.

Step 7 Repeat Step 4 to Step 6 until all the demands are satisfied and all the supplies are exhausted.

Step 8 This allotment yields a fuzzy initial basic feasible solution to the given fuzzy transportation problem.
i.e., Total fuzzy cost $=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \quad x_{ij}.$

6 Numerical Example

Consider the transportation problem with three origins $O_i (1 \le i \le 3)$ and four destinations $D_j\ (1 \le j \le 4)$ with the fuzzy transportation cost for unit quantity of transporting the product from O_i to D_j as

$$[\tilde{C}_{ij}] = \begin{bmatrix} (6,7,15,17;0.5) & (5,12,19,20;0.6) & (5,10,15,20;1) & (1,2,2,3;0.7) \\ (1,10,13,20;1) & (10,20,30,40;0.7) & (4,16,25,36;1) & (2,3,6,8;0.5) \\ (4,8,32,64;0.5) & (1,8,27,64;0.6) & (15,17,19,21;0.9) & (1,8,19,20;0.6) \end{bmatrix}$$

The crisp availability of the product at origins and destinations are 50, 50, 50 and 30,40,55,25 respectively.
The fuzzy transportation problem is,
where $\sum a_i = \sum b_j$

	D_1	D_2	D_3	D_5	Supply
O_1	(6,7,15,17;0.5)	(5,12,19,20;0.6)	(5,10,15,20;1)	(1,2,2,3;0.7)	50
O_2	(1,10,13,20;1)	(10,20,30,40;0.7)	(4,16,25,36;1)	(2,3,6,8;0.5)	50
O_3	(4,8,32,64;0.5)	(1,8,27,64;0.6)	(15,17,19,21;0.9)	(1,8,19,20;0.6)	50
Demand	30	40	55	25	

Now we find the rank value of each cell by using the proposed (apex-base angle) method,

$R(\tilde{C}_{11}) = 5$ $R(\tilde{C}_{12}) = 9$ $R(\tilde{C}_{13}) = 13$ $R(\tilde{C}_{14}) = 2$
$R(\tilde{C}_{21}) = 11$ $R(\tilde{C}_{22}) = 18$ $R(\tilde{C}_{23}) = 20$ $R(\tilde{C}_{24}) = 3$
$R(\tilde{C}_{31}) = 14$ $R(\tilde{C}_{32}) = 15$ $R(\tilde{C}_{33}) = 16$ $R(\tilde{C}_{34}) = 7$

Now, we replace these values for their corresponding $\tilde{C}_{ij}$ in which result in a convenient balanced transportation

TABLE 1

	D_1	D_2	D_3	D_4	Supply
O_1	5	9	13	2	50
O_2	11	18	20	3	50
O_3	14	15	16	7	50
Demand	30	40	55	25	

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Now, using the step 3 to step 7 of the ABLC method, we have the following reduced transportation tables,

TABLE 2

					Supply
	5	9	13	2	50
	11	18	20	3 25	50
	14	15	16	7	50
Demand	30	40	55	**25**	

TABLE 3

				Supply
	5	9 40	13	50
	11	18	20	50(25)
	14	15	16	50
Demand	**30**	40	55	

TABLE 4

			Supply
	5	13	50(10)
	11 25	20	50(25)
	14	16	50
Demand	**30**	55	

TABLE 5

			Supply
	5 5	13	50(10)
	14	16	**50**
Demand	30(5)	55	

TABLE 6

		Supply
	13 5	50(5)
	16 50	**50**
Demand	55	

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By applying ABLC method, the initial basic feasible solution is given in the following table.

TABLE 7

	D_1	D_2	D_3	D_4	Supply
O_1	5 5	9 40	13 5	2	50
O_2	11 25	18	20	3 25	50
O_3	14	15	16 50	7	50
Demand	30	40	55	25	

The corresponding initial transportation cost is given by
Z= (5 * 5) + (40 * 9) + (5 * 13) + (25 * 11) + (25 * 3) + (50 * 16)
= 1600

6.1 Comparison of proposed method with existing methods

The initial basic feasible solution obtained using the existing methods are given below

Existing methods	Proposed method
1.North West Corner rule	**ABLC Method**
$x_{11} = 30,\quad x_{12} = 20,$ $x_{22} = 20,\quad x_{23} = 30$ $x_{33} = 25\quad x_{34} = 25$ Total cost = 1865	$x_{11} = 5,\quad x_{12} = 40$ $x_{13} = 5,\quad x_{21} = 5$ $x_{24} = 25\,,\quad x_{33} = 50$ Total cost = 1600

2.Matrix-minima
$x_{11} = 25,\ x_{14} = 25,$
$x_{21} = 5,\ x_{23} = 45$
$x_{32} = 40,\ x_{33} = 10$
Total cost = 1890

3. VAM's Method
$x_{11} = 5,\quad x_{12} = 40,$
$x_{13} = 5,\quad x_{21} = 25,$
$x_{24} = 25,\quad x_{33} = 50$
Total cost = 1600

4. Exponential approach
$x_{11} = 5,\quad x_{12} = 40$
$x_{13} = 5,\quad x_{21} = 25$
$x_{24} = 25,\quad x_{33} = 50$

Total cost = 1600

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